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Internal magnetic fields and supersymmetry in orientifolds

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Abstract

Within the context of type I strings, we show the equivalence between BPS D9 branes with internal magnetic fluxes H_i in the three torii and non-BPS D3 branes with inverted internal magnetic fluxes $1/H_i$. We then construct new supersymmetric examples of $Z_2 \times Z_2$ orientifolds with discrete torsion which in the past had only non-supersymmetric solutions and emphasize the role of new twisted tadpole cancellation conditions, arising in the presence of magnetic fields, in order to get a consistent spectrum. In a second and independent part of the paper, we construct a new nine-dimensional type IIB orientifold with Scherk-Schwarz deformation which has the peculiarity of introducing a new type of non-BPS $O9$ planes and which contains as top branes a Scherk-Schwarz deformation of non-BPS D9 branes. The model contains charged D7 and D3 branes with a soft supersymmetry breaking spectrum.

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1. Introduction

Internal magnetic fluxes in string theory first proved their interesting features by providing the an explicit realization of the non-linear Born-Infeld electrodynamics [1, 2]. One of its simplest consequences is that, since magnetic fields couple differently to particles of different spins, they naturally break supersymmetry [3], a notoriously difficult problem in string theory. Complete string theory vacua with $\mathcal{N} = 1$ supersymmetry or no supersymmetry at all were constructed some time ago in the T-dual picture of branes intersecting at angles [4] or in the internal magnetic fields description [5]. A collective effort over the last years did lead to models of particle physics closer and closer to the Standard Model or its minimal supersymmetric extension, at the same time providing new ways of implementing inflation in string theory [6]. One of the new interesting features of internal magnetic fluxes is that they induce RR charges and tensions for branes of different dimensionalities [7]. This fact, of crucial importance for finding consistent string vacua [5, 4], has also the welcome feature of allowing radical rank reductions for the gauge group and of allowing new supersymmetric orientifold vacua.

Our present paper was trigerred by the curious but by now well-established fact that some of the induced tensions on magnetized D-branes can be negative. This interesting observation does raise the hope of finding new supersymmetric solutions in situations where this was regarded as impossible in the past, especially in models featuring the phenomenon of "brane supersymmetry breaking" [8, 9]. Indeed, this issue was recently raised in [10, 11] in a particular class of constructions and [11] did display non-chiral supersymmetric $Z_2 \times Z_2$ orientifold constructions in this framework, using both BPS and non-BPS bulk D-branes with magnetic fluxes in their worldvolume. The goal of the present paper is a more detailed understanding of the conditions under which supersymmetric constructions do exist, to set the general consistency rules and to find the massless spectrum for this new type of supersymmetric $Z_2 \times Z_2$ orientifold constructions with discrete torsion and branes at the orbifold fixed points.

The organization of the paper is as follows. In Section 2 we show that BPS and non-BPS D-branes in type I strings in the presence of internal magnetic fluxes are fully equivalent at the string level by performing a simple mapping of the fluxes from the BPS to the non-BPS branes. In Section 3 we present general considerations and necessary conditions for obtaining supersymmetric constructions with three internal magnetic fields on an arbitrary number of stacks of D-branes, which forces upon specific orbifold constructions. Section 4

presents our main new results containing new chiral supersymmetric $Z_2 \times Z_2$ orientifolds with discrete torsion with D-branes at orbifold fixed points. In this case, we also show that the twisted RR tadpole conditions add new consistency constraints compared to the corresponding orientifolds without magnetic fields. We present the general pattern of the gauge group and of the massless spectrum and give explicit examples. Section 5, independent of the rest of the paper, contains a new orientifold of Scherk-Schwarz type II string compactifications in nine dimensions, which has the new feature of generating non-BPS orientifold O9 planes of a new type. The top D9 branes in the model are a soft Scherk-Schwarz deformation of non-BPS branes of type I strings, whereas the model contains charged D7 and D3 branes with a soft supersymmetry breaking spectrum. Finally, we present our conclusions and present in a short appendix some useful formulae and conventions.

2. Non-BPS versus BPS brane transmutation in type I strings

The type I string has stable BPS D9 and D5 branes (also D1 which will play no role in our discussion), whereas all its other Dp branes are unstable. Of particular interest for our present discussion are, however, the non-BPS D7 and D3 branes, which were constructed by Sen [12] starting from Type II D-branes and consistently imposing the orientifold projection. Taking into account that the type I orientifold projection antisymmetrizes the RR 8-form and 4-form fields coupling to D3 and D7 branes, the construction starts from an equal pair $M = N$ of brane-antibrane pairs in Type IIB, which are interchanged by the orientifold projection. The procedure creates a new stack of uncharged branes, with gauge group $U(D)$, where $D = M = N$, obtained from the original gauge group $U(M) \times U(N)$ by a suitable identification of the Chan-Paton charges. The CFT description of a stack of non-BPS D3 and D7 branes was presented in [13].

A simple and popular way of breaking partly or totally supersymmetry is by adding magnetic fluxes on the BPS branes [3, 4, 5]. Considering for simplicity a factorizable internal space $T^2 \times T^2 \times T^2$ of coordinates $w_i = x_i + iy_i$ and volumes v_i, we will denote the internal magnetic fields on D9 branes pointing in the three different torii by H_1, H_2, H_3. The three magnetic fields then satisfy a generalized version of the Dirac quantization condition

$$H_i = \frac{m_i}{n_i v_i} \ , \tag{2.1}$$

where (m_i, n_i) are integers generalizing the Landau levels of a particle in a magnetic field in quantum mechanics. By performing 3 T-dualities in the coordinates x_i, one in each

compact torus T_i^2, we can trade the magnetic fluxes for D6-brane rotations

$$\tan\theta_i = H_i \ , \tag{2.2}$$

where θ_i is the angle in the torus T_i^2 made by the D6 brane with the coordinate y_i. Arbitrary rotations in the three tori break completely supersymmetry. For particular cases, however, partial supersymmetry is restored. For example, $\theta_3 = 0$ and $\theta_1 \pm \theta_2 = 0$ leaves unbroken $\mathcal{N} = 2$ supersymmetry from a four-dimensional viewpoint, whereas

$$\theta_1 \pm \theta_2 \pm \theta_3 = 0 \tag{2.3}$$

with all angles different from zero preserves $\mathcal{N} = 1$ supersymmetry in toroidal compactifications. By using (2.1), the conditions (2.3) translate into non-linear conditions for the internal magnetic fields, which for the $(+,+)$ sign in (2.3), for example, is

$$H_1 + H_2 + H_3 = H_1 H_2 H_3 \ . \tag{2.4}$$

The RR charges of various fields are given by the Wess-Zumino terms

$$\begin{aligned}
&\frac{n_1 n_2 n_3}{2}\left(q_9 \int_{\mathcal{M}_{10}} C \wedge e^{s_1 H_1 + s_2 H_2 + s_3 H_3} + q_9 \int_{\mathcal{M}_{10}} C \wedge e^{-s_1 H_1 - s_2 H_2 - s_3 H_3} \right) \\
&= n_1 n_2 n_3 q_9 \ \int_{\mathcal{M}_{10}} C_{10} + \\
&n_1 n_2 n_3 q_9 \ \int_{\mathcal{M}_{10}} C_6 \wedge (s_1 s_2 H_1 \wedge H_2 + s_2 s_3 H_2 \wedge H_3 + s_1 s_2 H_1 \wedge H_3)
\end{aligned} \tag{2.5}$$

where $s_i/2 = \pm 1/2$, $i = 1,2,3$ are the internal helicities of the (left or right from the worldsheet point of view) fermions. The terms in (2.5) correspond in the effective action to the RR couplings

$$\begin{aligned}
&n_1 n_2 n_3 q_9 \int_{\mathcal{M}_{10}} C_{10} + q_5 \left[s_2 s_3 n_1 m_2 m_3 \int_{\mathcal{M}_6^{(1)}} C_6^{(1)} + s_1 s_3 m_1 n_2 m_3 \int_{\mathcal{M}_6^{(2)}} C_6^{(2)} \right. \\
&\left. + s_1 s_2 m_1 m_2 n_3 \int_{\mathcal{M}_6^{(3)}} C_6^{(3)} \right]
\end{aligned} \tag{2.6}$$

where q_9 is the charge of one D9 brane, q_5 is the charge of one D5 brane and $C_6^{(i)}$ are six-form RR fields coupling to $D5_i$ branes with worldvolumes wrapping tori T_i^2. Therefore the brane acquires lower-dimensional charges, as if it were to contain lower-dimensional branes

[7]. By using the quantization conditions (2.1), these are precisely induced charges of the D5 branes type in the internal space which will modify tadpole consistency conditions [5].

On the other hand, for arbitrary rotation angles the effective tension of the BPS branes with fluxes cannot be understood in terms of the original tension plus induced lower dimensional tensions. Indeed, the tension of the magnetized D9 branes, corresponding in character language to the brane coupling to the closed string field $V_2O_2O_2O_2$ in the tree-level channel cylinder amplitude, is

$$T \ = \ |n_1n_2n_3|T_9 \sqrt{(1+H_1^2)(1+H_2^2)(1+H_3^2)} \ , \tag{2.7}$$

where $T_9 = q_9$ is the tension of a single D9 brane. Eq. (2.7), combined with the quantization condition (2.1), cannot be given generally the same simple and elegant interpretation as the RR charge (2.5). In the particular case where the supersymmetry condition is satisfied, however, this becomes possible and matches precisely the RR charge interpretation. For example, in the case $\theta_3 = 0$ and $\theta_1 \pm \theta_2 = 0$, (2.1) gives the constraint $(m_1/n_1v_1) = \pm(m_2/n_2v_2)$ and the induced tension reads $T = T_9 \ (|n_1n_2| + \frac{|m_1m_2|}{v_1v_2}) = |n_1n_2| \ T_9 + |m_1m_2|T_5$, corresponding in the effective theory in the string frame to the terms

$$|n_1n_2| \ T_9 \int d^{10}x \ \sqrt{g} \ e^{-\phi} \ + |m_1m_2|T_5 \int d^6x \ \sqrt{g} \ e^{-\phi} \ , \tag{2.8}$$

where T_5 is the tension of a single D5 brane. In this case, the induced tension is always positive and the magnetic fluxes can mimic D5 brane or antibrane tensions and charges [5]. In the case of three rotations preserving $\mathcal{N} = 1$ supersymmetry, the situation is similar, with one important difference. For the $(+,+)$ sign in (2.3), for example and combining (2.4) with (2.7), we find

$$\begin{aligned} T_{\rm eff} \ &= \ |n_1n_2n_3 \ T_9 \ (1 - H_1H_2 - H_2H_3 - H_1H_3)| \ = \ |n_1n_2n_3 \ T_9 \\ &-T_5 \ (n_1m_2m_3 + \ m_1n_2m_3 + \ m_1m_2n_3) \ sgn(n_1n_2n_3)| \ . \end{aligned} \tag{2.9}$$

Similarly, the couplings of the three internal volume (Kahler) fields $O_2V_2O_2O_2$, $O_2O_2V_2O_2$ and $O_2O_2O_2V_2$, obtained factorizing open string amplitudes in the tree-level closed channel, using (2.4) can be shown to be

$$|n_1n_2n_3|T_9 \ (1-H_1^2)\sqrt{\frac{(1+H_2^2)(1+H_3^2)}{(1+H_1^2)}} = |n_1n_2n_3|T_9(1+H_1H_2-H_2H_3+H_1H_3) \ ,$$

$$|n_1n_2n_3|T_9 \ (1-H_2^2)\sqrt{\frac{(1+H_1^2)(1+H_3^2)}{(1+H_2^2)}} = |n_1n_2n_3|T_9(1+H_1H_2+H_2H_3-H_1H_3) \ ,$$

$$|n_1n_2n_3|T_9\ (1-H_3^2)\sqrt{\frac{(1+H_1^2)(1+H_2^2)}{(1+H_3^2)}}=|n_1n_2n_3|T_9(1-H_1H_2+H_2H_3+H_1H_3).\tag{2.10}$$

There is in this case a correct match of the various tensions in (2.9) with the various RR charges in (2.5), signaling spacetime supersymmetry in the spectrum. The qualitative difference compared with the previous case (2.8) is that now it is possible that one of the three induced tensions in (2.9) be negative, as also emphasized recently in [14].

In orbifold constructions with internal fluxes, twisted RR tadpole conditions can also induce new non-trivial constraints, as we will see in explicitly for the case of $Z_2 \times Z_2$ orientifold with discrete torsion.

We now turn to type I non-BPS branes with fluxes. Let us consider the type I non-BPS D3 branes in a compact space setup and perform six T-dualities. The result are the non-BPS D9 branes in the type IIB orientifold $\Omega' = \Omega I_6 (-1)^{F_L}$, where the orientifold projection identifies the D9-D$\bar{9}$ pairs of the type IIB string. In this case, the conditions for having $\mathcal{N}=1$ supersymmetry are[1]

$$\phi_1+\phi_2+\phi_3=\frac{\pi}{2}\ ,\tag{2.11}$$

where ϕ_i are rotation angles for the non-BPS branes. Analogously to (2.4), (2.11) translates into a non-linear condition for the internal magnetic fields

$$\mathcal{H}_1\mathcal{H}_2+\mathcal{H}_2\mathcal{H}_3+\mathcal{H}_1\mathcal{H}_3=1\ ,\tag{2.12}$$

where $\mathcal{H}_i = m_i'/(n_i'v_i')$ are magnetic fluxes added on the non-BPS D9 branes. Interestingly, even if the non-BPS D9 branes have no RR charges, magnetic fluxes will induce lower dimensional ones, which can be found starting from the D9-D$\bar{9}$ pairs of type IIB

$$\begin{aligned}&\frac{n_1'n_2'n_3'}{2}\ \Big(\ q_9\int_{\mathcal{M}_{10}}C\wedge\ e^{s_1\mathcal{H}_1+s_2\mathcal{H}_2+s_3\mathcal{H}_3}-q_9\int_{\mathcal{M}_{10}}C\wedge\ e^{-s_1\mathcal{H}_1-s_2\mathcal{H}_2-s_3\mathcal{H}_3}\ \Big)\\&=n_1'n_2'n_3'\ q_9\ \int_{\mathcal{M}_{10}}C_8\wedge\ (s_1\mathcal{H}_1+s_2\mathcal{H}_2+s_3\mathcal{H}_3)\\&+\prod_{i=1}^{3}(s_in_i')\ q_9\ \int_{\mathcal{M}_{10}}C_4\wedge\mathcal{H}_1\wedge\mathcal{H}_2\wedge\mathcal{H}_3\end{aligned}\tag{2.13}$$

corresponding in the effective action to the RR couplings

$$\prod_{i=1}^{3}(s_im_i')q_3\int_{\mathcal{M}_4}\ C_4+$$

[1] As for the BPS brane case, there are several sign choices in toroidal compactifications. We choose for definiteness the $(+,+,+)$ signs.

$$q_7 \left[s_1 m_1' n_2' n_3' \int_{\mathcal{M}_8^{(1)}} C_8^{(1)} + s_2 n_1' m_2' n_3' \int_{\mathcal{M}_8^{(2)}} C_8^{(2)} + s_3 n_1' n_2' m_3' \int_{\mathcal{M}_8^{(3)}} C_8^{(3)} \right] \quad (2.14)$$

where C_4 is the four form coupling to D3 branes and $C_8^{(i)}$ are the eight-forms couplings to three different types of D7 branes. The tension of non-BPS branes is similar to (2.7), after a rescaling, so that $\mathcal{T}_9 = \sqrt{2} T_9$. Then in the supersymmetric case (2.11), combining the analog of (2.7) with (2.11), we find the non-BPS brane tension to be given by

$$\begin{aligned} \mathcal{T} &= |n_1' n_2' n_3' \mathcal{T}_9 \ (\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 - \mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3)| \\ &= |m_1' m_2' m_3' \ \mathcal{T}_3 + \mathcal{T}_7 (m_1' n_2' n_3' \ + \ n_1' m_2' n_3' \ + \ n_1' n_2' m_3')| \ . \end{aligned} \quad (2.15)$$

This expression has the interesting property of having interpretation in terms of purely lower dimensional tensions, while the original tension has disappeared ! It also matches the RR charges (2.13), justifying the presence of supersymmetry in the spectrum.

Since adding arbitrary magnetic fluxes erases the main difference between BPS and non-BPS branes, one can wonder whether a precise mapping exists between BPS branes with fluxes H_i and non-BPS branes with fluxes $\mathcal{H}_i$, a mapping $H_i \leftrightarrow \mathcal{H}_i$ under which (2.4) turns into (2.12) with the same spectrum on both BPS and non-BPS branes. Interestingly enough, we find that such a transformation exists and is simply given by

$$H_i \quad \leftrightarrow \quad \frac{1}{\mathcal{H}_i} \ . \quad (2.16)$$

The interpretation of (2.16) is that the BPS D9 branes with fluxes (m_i, n_i) satisfying the condition (2.3)-(2.4) are mapped after six T-dualities $v_i \leftrightarrow 1/v_i'$ into non-BPS D3 branes with fluxes $(m_i', n_i') = (n_i, m_i)$, satisfying (2.11)-(2.12). The reason behind (2.16) is actually easy to understand. Let us start with a stack of M non-BPS D3 branes in type I. The construction starting from type II branes and the spectrum, pioneered in [12], were described in detail in [13]. The open string amplitudes in the presence of internal magnetic fields are modified to[2]

$$\mathcal{A} = \int_0^\infty \frac{dt}{t^3} \Big\{ M \bar{M} \ \tilde{P}_1 \tilde{P}_2 \tilde{P}_3 (V_2 O_6 + O_2 V_6 - S_2 S_6 - C_2 C_6)(0) \frac{1}{\eta^6(\tau)} + \prod_{i=1}^{3} (2 n_i m_i) \times$$
$$\left[\frac{M^2}{2} \ (O_2 O_6 + V_2 V_6 - S_2 C_6 - C_2 S_6)(2 z_i' \tau) + \frac{\bar{M}^2}{2} \ (O_2 O_6 + V_2 V_6 - S_2 C_6 - C_2 S_6)(-2 z_i' \tau) \right]$$

[2]For the notations and conventions see. e.g. the last two review papers in [15] and the appendix of this paper.

$$\times \prod_{i=1}^{3} \frac{i\eta}{\vartheta_1(2z_i'\tau)} \Big\} \frac{1}{\eta^2(\tau)} \ ,$$

$$\mathcal{M} = \int_0^\infty \frac{dt}{t^3} \prod_{i=1}^{3} (2m_i) \Big\{ \Big[\frac{M}{2} (\hat{O}_2\hat{O}_6 + \hat{V}_2\hat{V}_6)(2z_i'\tau) + \frac{\bar{M}}{2} (\hat{O}_2\hat{O}_6 + \hat{V}_2\hat{V}_6)(-2z_i'\tau) \Big]$$

$$- \Big[\frac{M}{2} (\hat{S}_2\hat{C}_6 - \hat{C}_2\hat{S}_6)(2z_i'\tau) - \frac{\bar{M}}{2} (\hat{S}_2\hat{C}_6 - \hat{C}_2\hat{S}_6)(-2z_i'\tau) \Big] \Big\} \frac{1}{\eta^2(\tau)} \prod_{i=1}^{3} \frac{i\eta}{\vartheta_1(2z_i'\tau)} \ , \qquad (2.17)$$

where $\phi_i \equiv \pi z_i'$ and where $\tilde{P}_i$ denote "boosted" compactification lattices [2] obtained replacing the Kaluza-Klein momenta $\mathbf{k}_i$ in the ith torus by $\mathbf{k}_i \to \mathbf{k}_i/\sqrt{n_i^2 + (m_i^2/v_i^2)}$. Then by using the identities (7.1) for the Jacobi functions, it is straightforward to show that (2.17) transform precisely into the amplitudes of the BPS D9 branes in type I with magnetic fluxes

$$z_i = z_i' + \frac{1}{2} \ , \qquad (2.18)$$

after 6 T-dualities. The mapping (2.18) implies (2.16) and in particular explains, through the additional $\pi/2$ rotations in each torus, the change of the GSO projection in going from the non-BPS branes in (2.17) to the BPS branes. Therefore, our results show that the examples proposed in [10, 11] with magnetic fluxes on non-BPS branes can be entirely described by appropriately modified fluxes on BPS branes. In section 4 we describe new supersymmetric solutions of the $Z_2 \times Z_2$ orientifold with discrete torsion obtained using only BPS branes with internal magnetic fluxes.

3. Supersymmetry and negative induced tensions : general considerations

At first sight, the internal fluxes on the BPS D9 branes induce D5_i type tensions (2.9), which are of two distinct types

$$(+,+,-) \text{ and permutations} \quad \text{if} \quad H_1, H_2 > 0 \ , \ H_3 < 0 \ ,$$
$$(-,-,-) \quad \text{if} \quad H_1, H_2, H_3 > 0 \ , \qquad (3.1)$$

and the reversed solutions $H_i \to -H_i$. The second case is incompatible, however, with the supersymmetry condition (2.3). Notice that it is not possible to generate only positive induced D5_i type tensions. Let us consider in the following p stacks of D9 branes M_a, $a = 1 \cdots p$. The $\mathcal{N} = 1$ supersymmetry conditions (2.4) on each stack and the D9 and D5

tadpole conditions in toroidal, Z_2 and $Z_2 \times Z_2$ orbifold compactifications can be written compactly as

$$
\begin{aligned}
& m_1^{(a)} n_2^{(a)} n_3^{(a)} v_2 v_3 + n_1^{(a)} m_2^{(a)} n_3^{(a)} v_1 v_3 + n_1^{(a)} n_2^{(a)} m_3^{(a)} v_1 v_2 = \prod_{i=1}^{3} m_i^{(a)} \quad , \quad \text{for any a} \\
& \sum_a M_a n_1^{(a)} n_2^{(a)} n_3^{(a)} = 16 \ , \\
& \sum_a M_a n_1^{(a)} m_2^{(a)} m_3^{(a)} = -16 \ \epsilon_1 \ , \\
& \sum_a M_a m_1^{(a)} n_2^{(a)} m_3^{(a)} = -16 \ \epsilon_2 \ , \\
& \sum_a M_a m_1^{(a)} m_2^{(a)} n_3^{(a)} = -16 \ \epsilon_3 \ ,
\end{aligned} \tag{3.2}
$$

where

$$
\begin{aligned}
& (\epsilon_1, \epsilon_2, \epsilon_3) = (0,0,0) \quad \text{in toroidal comp. ,} \\
& (\epsilon_1, \epsilon_2, \epsilon_3) = (\pm 1,0,0) \quad \text{in } Z_2 \text{ comp. ,} \\
& (\epsilon_1, \epsilon_2, \epsilon_3) = (\pm 1,\pm 1,\pm 1) \quad \text{in } Z_2 \times Z_2 \text{ comp. .}
\end{aligned} \tag{3.3}
$$

Defining $n^{(a)} \equiv n_1^{(a)} n_2^{(a)} n_3^{(a)}$, the tadpole conditions in (3.2) can also be written

$$
\begin{aligned}
& \sum_a M_a n^{(a)} = 16 \ , \\
& \sum_a M_a n^{(a)} H_2^{(a)} H_3^{(a)} = -16 \ \frac{\epsilon_1}{v_2 v_3} \ , \\
& \sum_a M_a n^{(a)} H_1^{(a)} H_3^{(a)} = -16 \ \frac{\epsilon_2}{v_1 v_3} \ , \\
& \sum_a M_a n^{(a)} H_1^{(a)} H_2^{(a)} = -16 \ \frac{\epsilon_3}{v_1 v_2} \ .
\end{aligned} \tag{3.4}
$$

Let us consider in the following the case with all $n_i^{(a)} > 0$. Combining (3.4) with (2.9), we immediately find

i) if

$$
1 - \sum_{i<j} H_i^{(a)} H_j^{(a)} \ > \ 1 \ , \tag{3.5}
$$

then

$$
\frac{\epsilon_1}{v_2 v_3} + \frac{\epsilon_2}{v_1 v_3} + \frac{\epsilon_3}{v_1 v_2} \ > \ 0 \ . \tag{3.6}
$$

Therefore, in this case the toroidal compactifications[3] and the T^4/Z_2 orientifold example with exotic $\mathrm{O5}_-$ planes are excluded (they are incompatible with supersymmetry), the $Z_2 \times Z_2$ orientifold without discrete torsion $\epsilon_i = (1,1,1)$ is possible, whereas for the other cases necessary (but not sufficient) conditions are provided by

$$\begin{aligned} &\epsilon_i = (1,1,-1) \text{ possible if } v_1 + v_2 \ > \ v_3 \ , \\ &\epsilon_i = (1,-1,-1) \text{ possible if } v_1 \ > \ v_2 + v_3 \ . \end{aligned} \tag{3.7}$$

ii) if

$$1 - \sum_{i<j} H_i^{(a)} H_j^{(a)} \ < \ -1 \ , \tag{3.8}$$

then

$$\frac{\epsilon_1}{v_2 v_3} + \frac{\epsilon_2}{v_1 v_3} + \frac{\epsilon_3}{v_1 v_2} \ < \ -2 \ . \tag{3.9}$$

In this case the toroidal constructions are again impossible, as well as the standard T^4/Z_2 orientifold with $\mathrm{O5}_+$ planes and the $Z_2 \times Z_2$ orientifold without discrete torsion $\epsilon_i = (1,1,1)$. For the other cases, there are conditions analogous to (3.7) which we do not explicitly display for brevity.

4. Chiral supersymmetric $Z_2 \times Z_2$ models with discrete torsion

We now turn to four dimensional compactification on $T^2 \times T^2 \times T^2$ of the Type-IIB theory, orbifolded by the $Z_2 \times Z_2$ action generated by the identity (we will call it "o") and the π rotations $g : (+,-,-)$, $f : (-,+,-)$, $h : (-,-,+)$, where the three entries within the parentheses refer to the three internal tori, while "+" and "−" are the two group elements of Z_2.

There are several choices actually, depending on three signs $\epsilon_i = \pm 1$, where $\epsilon_i = 1$ typically signals the existence of $O5_+$ planes and $\epsilon_i = -1$ that of $O5_-$ planes. The different possibilities are restricted by the condition

$$\epsilon \quad = \quad \epsilon_1 \ \epsilon_2 \ \epsilon_3 \ . \tag{4.1}$$

The case $\epsilon = 1$ in (4.1) defines models without discrete torsion, whereas the case $\epsilon = -1$ in (4.1) defines models with discrete torsion. The simplest model without discrete torsion

[3]Supersymmetric toroidal examples were recently provided in [14], in the case of non-zero off-diagonal fluxes between the internal tori. We do not consider generalized constructions of this type here.

$\epsilon_i = (1,1,1)$ was worked out in [16, 17] . It was proposed some time ago that all the other models, having at least one $\epsilon_1 = -1$, have no supersymmetric solution. The reason advanced in [9] is that $O5_{i,-}$ planes have positive tension and positive charge and would ask for a supersymmetric solution for compensating negative tension and charge, impossible to obtain by adding branes. Therefore RR tadpole conditions ask for the introduction of $\overline{D5}_i$ antibranes and supersymmetry is necessarily broken. This conclusion, as emphasized recently in [11], should be revised in models with three internal magnetic fields backgrounds, according to the possibility of having one negative induced tension in (2.9). In contrast to [10, 11], however, we consider here fractional D-branes, i.e. branes coupling to twisted sector fields. One possible advantage from a model building point of view in these models compared to the (simpler) ones without discrete torsion is that in all $Z_2 \times Z_2$ orientifold models without discrete torsion constructed in the literature there are three chiral (super)fields in the adjoint representation of the gauge group, an unwanted feature for phenomenological applications. On the other hand, the bulk branes considered in [11] lead to non-chiral spectra, whereas fractional branes allow in our constructions to get chiral spectra. The closed string amplitudes for all $Z_2 \times Z_2$ orientifolds are described by the torus amplitude[4]

$$\begin{aligned}
\mathcal{T} \;=\; & \int \frac{d^2\tau}{\tau_2^3}\frac{1}{4}\Big\{\Lambda_1\Lambda_2\Lambda_3|T_{oo}|^2 + \Lambda_1|T_{og}|^2|\frac{4\eta^2}{\vartheta_2^2}|^2 + \Lambda_2|T_{of}|^2|\frac{4\eta^2}{\vartheta_2^2}|^2 + \Lambda_3|T_{oh}|^2|\frac{4\eta^2}{\vartheta_2^2}|^2 \\
& + \Lambda_1|T_{go}|^2\frac{4\eta^2}{\vartheta_4^2}|^2 + \Lambda_1|T_{gg}|^2\frac{4\eta^2}{\vartheta_3^2}|^2 + |T_{fo}|^2\Lambda_2|\;\frac{4\eta^2}{\vartheta_4^2}|^2 + |T_{ff}|^2\Lambda_2|\;\frac{4\eta^2}{\vartheta_3^2}|^2 \\
& + \Lambda_3|T_{ho}|^2\frac{4\eta^2}{\vartheta_4^2}|^2 + \Lambda_3|T_{hh}|^2\frac{4\eta^2}{\vartheta_3^2}|^2 \\
& + \epsilon\;|\frac{8\eta^3}{\vartheta_2\vartheta_3\vartheta_4}|^2(|T_{gh}|^2 + |T_{gf}|^2 + |T_{hg}|^2 + |T_{hf}|^2 + |T_{fg}|^2 + |T_{fh}|^2)\Big\}\frac{1}{|\eta|^4}\;, \qquad (4.2)
\end{aligned}$$

and the Klein bottle amplitude

$$\begin{aligned}
\mathcal{K} \;=\; & \int_0^\infty \frac{d\tau_2}{\tau_2^3}\frac{1}{8}\Big\{(P_1P_2P_3 + P_1W_2W_3 + W_1P_2W_3 + W_1W_2P_3)T_{oo} + \\
& 32\left(\frac{\eta}{\vartheta_4}\right)^2[\epsilon_1(P_1+\epsilon W_1)T_{go} + \epsilon_2(P_2+\epsilon W_2)T_{fo} + \epsilon_3(P_3+\epsilon W_3)T_{ho}]\Big\}\frac{1}{\eta^2}\;. \qquad (4.3)
\end{aligned}$$

The simplest orientifold model without discrete torsion $\epsilon_i = (+,+,+)$ [16, 17] has 48 chiral multiplets from the twisted sector, whereas the example with $(+,-,-)$ has 16

[4]For definitions of $Z_2 \times Z_2$ characters, see e.g. [9].

chiral multiplets and 32 vector multiplets from the twisted sector. For the models with discrete torsion, both choices $(+,+,-)$ and $(-,-,-)$ yield a massless twisted closed string spectrum composed of 48 chiral multiplets. The crucial difference between the models without discrete torsion and the models with discrete torsion is that in the first case the twisted sector fields cannot couple to the D-branes and therefore, by the open-closed duality, the orbifold has no action on the Chan-Paton factors, whereas in the second case the branes can have couplings to twisted fields which ask for new, twisted RR tadpole conditions. The case of interest for us, satisfying (4.1) is, up to permutations, $(\epsilon_1, \epsilon_2, \epsilon_3) = (1, 1, -1)$. In this case, we have configurations of $O5_{1,+}$ planes, $O5_{2,+}$ and $O5_{3,-}$ planes. Due to the action of the orbifold operations on the Chan-Paton (CP) factors, the appropriate CP parametrization for the case under consideration for several stacks of fractional branes is

$$\begin{aligned} &M_{a,o} = \ p_a + q_a + \bar{p}_a + \bar{q}_a \ , \ M_{a,g} = i \ (p_a + q_a - \bar{p}_a - \bar{q}_a) \ , \\ &M_{a,f} = i \ (p_a - q_a - \bar{p}_a + \bar{q}_a) \ , \ M_{a,h} = \ p_a - q_a + \bar{p}_a - \bar{q}_a \ . \end{aligned} \tag{4.4}$$

In the corresponding models with discrete torsion but without the internal magnetic fields the twisted tadpole conditions read

$$\sum_a M_{a,g} = \sum_a M_{a,f} = \sum_a M_{a,h} = 0 \ , \tag{4.5}$$

which have the simple solution $p_a = q_a$ in order to cancel the couplings to the twisted RR fields, giving gauge groups of the form $\prod_a U(p_a)^2$. In the case with internal magnetic fields however, with the stack a experiencing the fluxes $(m_i^{(a)}, n_i^{(a)})$, the conditions $p_a = q_a$, while still necessary, are not sufficient anymore, since the new twisted RR tadpoles are

$$\begin{aligned} &\sum_a (p_a + q_a) \ m_1^{(a)} \ = \ 0 \ , \\ &\sum_a (p_a - q_a) \ m_2^{(a)} \ = \ 0 \ , \ \sum_a (p_a - q_a) \ m_3^{(a)} \ = \ 0, \end{aligned} \tag{4.6}$$

and therefore, in addition to $p_a = q_a$, we obtain the new condition

$$\sum_a (p_a + q_a) \ m_1^{(a)} \ = \ 0. \tag{4.7}$$

The reason behind this new condition is that the couplings of twisted six-dimensional RR fields $\mathcal{C}_6$ (e.g. $S_2 C_2 O_2 O_2$) to the magnetized D-branes are of the form

$$\begin{aligned} &\frac{n_1^{(a)}}{2} \left((p_a + q_a) \int_{\mathcal{M}_6} \mathcal{C} \wedge \ e^{s_1 H_1^{(a)}} - (\bar{p}_a + \bar{q}_a) \int_{\mathcal{M}_6} \mathcal{C} \wedge \ e^{-s_1 H_1^{(a)}} \right) \\ &= \frac{1}{v_1} s_1 \ (p_a + q_a + \bar{p}_a + \bar{q}_a) \ m_1^{(a)} \int_{\mathcal{M}_4} \mathcal{C}_4 \ . \end{aligned} \tag{4.8}$$

Similarly to the case of section 2, where unphysical RR couplings of the non-BPS branes were turned into physical couplings by the internal magnetic fields, unphysical couplings of twisted RR fields to the D-branes are turned into physical couplings to twisted four-forms, leading to the new condition (4.7).

The cylinder amplitude, for models with discrete torsion containing only magnetized D9 branes, is most conveniently separated into several pieces. The strings propagating from one stack a of branes to the same or its image a' are described by

$$\mathcal{A}^{aa,aa'} = \frac{1}{4}\int_0^\infty \frac{dt}{t^3}\sum_a \Big\{ |p_a+q_a|^2 \tilde{P}_1\tilde{P}_2\tilde{P}_3 (V_8-S_8)(0)\frac{1}{\eta^6} \tag{4.9}$$

$$+ \Big[|p_a+q_a|^2 \tilde{P}_1 T_{og}(0) + |p_a-q_a|^2 \tilde{P}_2 T_{of}(0) + |p_a-q_a|^2 \tilde{P}_3 T_{oh}(0) \Big] (\frac{2\eta}{\vartheta_2})^2$$

$$+ \prod_{i=1}^3 (2m_i^{(a)} n_i^{(a)}) \left[\frac{(p_a+q_a)^2}{2} (V_8-S_8)(2z_i^{(a)}\tau) + \frac{(\bar{p}_a+\bar{q}_a)^2}{2}(V_8-S_8)(-2z_i^{(a)}\tau)\right] \prod_{i=1}^3 \frac{i\eta}{\vartheta_1(2z_i^{(a)}\tau)}$$

$$- \left[\frac{(p_a+q_a)^2}{2} T_{og}(2z_i^{(a)}\tau) + \frac{(\bar{p}_a+\bar{q}_a)^2}{2} T_{og}(-2z_i^{(a)}\tau)\right] \left(\frac{2im_1^{(a)}n_1^{(a)}\eta}{\vartheta_1(2z_1^{(a)}\tau)}\right) \prod_{i=2,3} \frac{2\eta}{\vartheta_2(2z_i^{(a)}\tau)}$$

$$- \left[\frac{(p_a-q_a)^2}{2} T_{of}(2z_i^{(a)}\tau) + \frac{(\bar{p}_a-\bar{q}_a)^2}{2} T_{of}(-2z_i^{(a)}\tau)\right] \left(\frac{2im_2^{(a)}n_2^{(a)}\eta}{\vartheta_1(2z_2^{(a)}\tau)}\right) \prod_{i=1,3} \frac{2\eta}{\vartheta_2(2z_i^{(a)}\tau)}$$

$$+ \left[\frac{(p_a-q_a)^2}{2} T_{oh}(2z_i^{(a)}\tau) + \frac{(\bar{p}_a-\bar{q}_a)^2}{2} T_{oh}(-2z_i^{(a)}\tau)\right] \left(\frac{2im_3^{(a)}n_3^{(a)}\eta}{\vartheta_1(2z_3^{(a)}\tau)}\right) \prod_{i=1,2} \frac{2\eta}{\vartheta_2(2z_i^{(a)}\tau)} \Big\} \frac{1}{\eta^2} \ ,$$

whereas the strings stretched between the stack a and the stack b (and their corresponding images) are described by

$$\mathcal{A}^{ab,ab'} = \frac{1}{4}\int_0^\infty \frac{dt}{t^3}\sum_{a\neq b} \Big\{ I^{ab} \left[(p_a+q_a)(\bar{p}_b+\bar{q}_b)(V_8-S_8)(z_i^{(ab)}\tau) + \text{h.c.}\right] \prod_{i=1}^3 \frac{i\eta}{\vartheta_1(z_i^{(ab)}\tau)}$$

$$+ I^{ab'} \left[(p_a+q_a)(p_b+q_b)(V_8-S_8)(z_i^{(ab')}\tau) + \text{h.c.}\right] \prod_{i=1}^3 \frac{i\eta}{\vartheta_1(z_i^{(ab')}\tau)} \tag{4.10}$$

$$+ \left[I_1^{ab} (p_a+q_a)(\bar{p}_b+\bar{q}_b)\ T_{og}(z_i^{(ab)}\tau) + \text{h.c.}\right] \left(\frac{i\eta}{\vartheta_1(z_1^{(ab)}\tau)}\right) \prod_{i=2,3} \frac{2\eta}{\vartheta_2(z_i^{(ab)}\tau)}$$

$$- \left[I_1^{ab'} (p_a+q_a)(p_b+q_b)\ T_{og}(z_i^{(ab')}\tau) + \text{h.c.}\right] \left(\frac{i\eta}{\vartheta_1(z_1^{(ab')}\tau)}\right) \prod_{i=2,3} \frac{2\eta}{\vartheta_2(z_i^{(ab')}\tau)}$$

$$+\left[I_2^{ab}(p_a-q_a)(\bar{p}_b-\bar{q}_b)\ T_{of}(z_i^{(ab)}\tau)+\text{h.c.}\right]\left(\frac{i\eta}{\vartheta_1(z_2^{(ab)}\tau)}\right)\prod_{i=1,3}\frac{2\eta}{\vartheta_2(z_i^{(ab)}\tau)}$$

$$-\left[I_2^{ab'}(p_a-q_a)(p_b-q_b)\ T_{of}(z_i^{(ab')}\tau)+\text{h.c.}\right]\ \left(\frac{i\eta}{\vartheta_1(z_2^{(ab')}\tau)}\right)\prod_{i=1,3}\frac{2\eta}{\vartheta_2(z_i^{(ab')}\tau)}$$

$$+\left[I_3^{ab}(p_a-q_a)(\bar{p}_b-\bar{q}_b)\ T_{oh}(z_i^{(ab)}\tau)+\text{h.c.}\right]\left(\frac{i\eta}{\vartheta_1(z_3^{(ab)}\tau)}\right)\prod_{i=1,2}\frac{2\eta}{\vartheta_2(z_i^{(ab)}\tau)}$$

$$+\left[I_3^{ab'}(p_a-q_a)(p_b-q_b)\ T_{oh}(z_i^{(ab')}\tau)+\text{h.c.}\right]\ \left(\frac{i\eta}{\vartheta_1(z_3^{(ab')}\tau)}\right)\prod_{i=1,2}\frac{2\eta}{\vartheta_2(z_i^{(ab')}\tau)}\Bigg\}\frac{1}{\eta^2}\ ,$$

where we defined the intersection numbers of the magnetized D9 branes of the stacks a and b (b') in the ith torus

$$I_i^{ab}=m_i^{(a)}n_i^{(b)}-n_i^{(a)}m_i^{(b)}\ ,\ I_i^{ab'}=m_i^{(a)}n_i^{(b)}+n_i^{(a)}m_i^{(b)}\ ,$$
$$I^{ab}=\prod_{i=1}^{3}I_i^{ab}\ ,\ I^{ab'}=\prod_{i=1}^{3}I_i^{ab'}\ , \tag{4.11}$$

and the effective fluxes on strings with one end on the stack a and the other end on the stack b (b')

$$z_i^{(ab)}=z_i^{(a)}-z_i^{(b)}\ ,\ z_i^{(ab')}=z_i^{(a)}+z_i^{(b)}\ . \tag{4.12}$$

The Mobius amplitude is

$$\mathcal{M}=-\int_0^\infty\frac{dt}{t^3}\Big\{\prod_{i=1}^{3}(m_i^{(a)})\left[(p_a+q_a)(\hat{V}_8-\hat{S}_8)(2z_i\tau)+(\bar{p}_a+\bar{q}_a)(\hat{V}_8-\hat{S}_8)(-2z_i^{(a)}\tau)\right]\prod_{i=1}^{3}\frac{i\hat{\eta}}{\hat{\vartheta}_1(2z_i\tau)}$$
$$-\epsilon_1\left[(p_a+q_a)\hat{T}_{og}(2z_i^{(a)}\tau)+(\bar{p}_a+\bar{q}_a)\hat{T}_{og}(-2z_i^{(a)}\tau)\right]\left(\frac{im_1^{(a)}\hat{\eta}}{\hat{\vartheta}_1(2z_1^{(a)}\tau)}\right)\prod_{i=2,3}\frac{n_i^{(a)}\hat{\eta}}{\hat{\vartheta}_2(2z_i^{(a)}\tau)}$$
$$-\epsilon_2\left[(p_a+q_a)\hat{T}_{of}(2z_i^{(a)}\tau)+(\bar{p}_a+\bar{q}_a)\hat{T}_{of}(-2z_i^{(a)}\tau)\right]\left(\frac{im_2^{(a)}\hat{\eta}}{\hat{\vartheta}_1(2z_2^{(a)}\tau)}\right)\prod_{i=1,3}\frac{n_i^{(a)}\hat{\eta}}{\hat{\vartheta}_2(2z_i^{(a)}\tau)}$$
$$-\epsilon_3\left[(p_a+q_a)\hat{T}_{oh}(2z_i^{(a)}\tau)+(\bar{p}_a+\bar{q}_a)\hat{T}_{oh}(-2z_i^{(a)}\tau)\right]\left(\frac{im_3^{(a)}\hat{\eta}}{\hat{\vartheta}_1(2z_3^{(a)}\tau)}\right)\prod_{i=1,2}\frac{n_i^{(a)}\hat{\eta}}{\hat{\vartheta}_2(2z_i^{(a)}\tau)}\ \Big\}\frac{1}{\eta^2}\ . \tag{4.13}$$

Transforming the various amplitudes (4.3),(4.9),(4.10),(4.13) in the tree-level closed channel and factorizing the resulting amplitudes, we obtain the untwisted RR tadpole cancellation conditions

$$\sum_a(p_a+q_a)\ n_1^{(a)}n_2^{(a)}n_3^{(a)}=16\quad ,$$

$$
\begin{aligned}
\sum_a (p_a+q_a)\ n_1^{(a)}m_2^{(a)}m_3^{(a)} &= -16\ \epsilon_1\ , \\
\sum_a (p_a+q_a)\ m_1^{(a)}n_2^{(a)}m_3^{(a)} &= -16\ \epsilon_2\ , \\
\sum_a (p_a+q_a)\ m_1^{(a)}m_2^{(a)}n_3^{(a)} &= -16\ \epsilon_3\ ,
\end{aligned} \tag{4.14}
$$

to be supplemented precisely by the twisted RR condition (4.7). We are interested in the following in supersymmetric models. Consequently the fluxes on the magnetized D9 branes will have to satisfy the supersymmetric condition (2.4) for each stack.

In order to display generically the massless spectrum in this class of models, let us define

$$
I^{aO} = 8\ \left(m_1^{(a)}m_2^{(a)}m_3^{(a)} - \epsilon_1 m_1^{(a)}n_2^{(a)}n_3^{(a)} - \epsilon_2 n_1^{(a)}m_2^{(a)}n_3^{(a)} - \epsilon_3 n_1^{(a)}n_2^{(a)}m_3^{(a)} \right)\ . \tag{4.15}
$$

Then the massless spectrum is based on the gauge group $\prod_a \mathbf{U}(\mathbf{p_a}) \otimes \mathbf{U}(\mathbf{q_a})$, with $p_a = q_a$ and chiral (super)fields in the representations

$$
\begin{array}{ll}
\textbf{Multiplicity} & \textbf{Representation} \\
1 & (\mathbf{p_a},\bar{\mathbf{q}}_\mathbf{a}) + (\bar{\mathbf{p}}_\mathbf{a},\mathbf{q_a})\ , \\
\frac{1}{8}\left(I^{aa'} + I^{aO} - 4I_1^{aa'} - 4I_2^{aa'} + 4I_3^{aa'}\right) & (\frac{\mathbf{p_a}(\mathbf{p_a}-\mathbf{1})}{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\frac{\mathbf{q_a}(\mathbf{q_a}-\mathbf{1})}{\mathbf{2}})\ , \\
\frac{1}{8}\left(I^{aa'} - I^{aO} - 4I_1^{aa'} - 4I_2^{aa'} + 4I_3^{aa'}\right) & (\frac{\mathbf{p_a}(\mathbf{p_a}+\mathbf{1})}{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\frac{\mathbf{q_a}(\mathbf{q_a}+\mathbf{1})}{\mathbf{2}})\ , \\
\frac{1}{4}\left(I^{aa'} - 4I_1^{aa'} + 4I_2^{aa'} - 4I_3^{aa'}\right) & (\mathbf{p_a},\mathbf{q_a})\ , \\
\frac{1}{4}\left(I^{ab'} - 4I_1^{ab'} - 4I_2^{ab'} + 4I_3^{ab'}\right) & (\mathbf{p_a},\mathbf{p_b}) + (\mathbf{q_a},\mathbf{q_b})\ , \\
\frac{1}{4}\left(I^{ab'} - 4I_1^{ab'} + 4I_2^{ab'} - 4I_3^{ab'}\right) & (\mathbf{p_a},\mathbf{q_b})\ , \\
\frac{1}{4}\left(I^{ab} + 4I_1^{ab} + 4I_2^{ab} + 4I_3^{ab}\right) & (\mathbf{p_a},\bar{\mathbf{p}}_\mathbf{b}) + (\mathbf{q_a},\bar{\mathbf{q}}_\mathbf{b})\ , \\
\frac{1}{4}\left(I^{ab} + 4I_1^{ab} - 4I_2^{ab} - 4I_3^{ab}\right) & (\mathbf{p_a},\bar{\mathbf{q}}_\mathbf{b})\ ,
\end{array} \tag{4.16}
$$

where above $a \neq b$ in order to avoid the overcounting of states. It is a straightforward exercise to show that the irreducible anomalies $SU(p_a)^3$ and $SU(q_a)^3$ with the spectrum (4.16) cancel precisely when the untwisted (4.14) and the twisted (4.7) RR cancellation conditions are satisfied. The other gauge anomalies are taken care by the generalized version of the four-dimensional Green-Schwarz mechanism [18].

IV-A. Explicit models

We can now provide examples of $Z_2 \times Z_2$ orientifolds with discrete torsion containing only magnetized D9 branes and no D5 branes. The first class of examples are those in which the twisted tadpole condition (4.7) is simply satisfied starting with a pair of magnetized stacks containing equal numbers of D9 branes with opposite magnetic fluxes. An explicit example in this case is $M_1 = M_2 = 8$ and internal magnetic fluxes

$$\begin{aligned}(m_i^{(1)}, n_i^{(1)}) &= (1,1),(1,1),(-1,1)\ ,\\ (m_i^{(2)}, n_i^{(2)}) &= (-1,1),(-1,1),(1,1)\ .\end{aligned} \tag{4.17}$$

Supersymmetry on each stack has actually several possible solutions in terms of the compact volumes. One possible solution corresponds to $(v_1, v_2, v_3) = (3,2,1)$ with the corresponding magnetic fluxes on the two stacks equal to $H^{(1)} = (1/3, 1/2, -1)$ and $H^{(2)} = (-1/3, -1/2, 1)$, respectively. The gauge group of the model is $\mathbf{U(4)}^2 \otimes \mathbf{U(4)}^2$, where we wrote separately the gauge factors $\mathbf{U(4)}^2$ coming from the first and from the second stack. The spectrum contains chiral multiplets in $(\mathbf{4}, \bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{4}, \bar{\mathbf{4}}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{4}}, \mathbf{4}) + 8 \times [(\mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{6}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \bar{\mathbf{6}})]$. In this example the twisted tadpole condition (4.7) is simply satisfied starting with a pair of stacks with opposite magnetic fluxes. It is easy to construct other similar examples of this type. We did indeed construct examples with gauge group $\mathbf{U(2)}^2 \otimes \mathbf{U(2)}^2$, $\mathbf{U(4)}^2 \otimes \mathbf{U(1)}^2$, and a third one based on the gauge group $\mathbf{U(2)}^2 \otimes \mathbf{U(1)}^2$.

Our second class of examples correspond to models in which the twisted tadpole condition (4.7) is satisfied starting with two stacks of different numbers of magnetized D9 branes and different compensating fluxes. An explicit example in this class is $M_1 = 8$, $M_2 = 4$, with internal magnetic fluxes

$$\begin{aligned}(m_i^{(1)}, n_i^{(1)}) &= (1,1),(1,1),(-1,1)\ ,\\ (m_i^{(2)}, n_i^{(2)}) &= (-2,2),(-1,1),(1,1)\ .\end{aligned} \tag{4.18}$$

A supersymmetric solution for the internal volumes and internal magnetic fields is the same as in the previous example. The gauge group of the model consists of two copies of the gauge group $\mathbf{U(4)}^2 \otimes \mathbf{U(2)}^2$. The chiral spectrum of the model is $8 \times [(\mathbf{6}, \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{6}, \mathbf{1}, \mathbf{1})] + 2 \times [(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})] + 12 \times (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})$, together with 36 $\mathbf{SU(4)}^2$ and $\mathbf{SU(2)}^2$ singlets charged under various $\mathbf{U(1)}$ factors.

A chiral model with a Standard Model type gauge group

Phenomenologically more interesting models can be constructed out of three stacks of magnetized D9 branes. An explicit example we found is based on three stacks containing six, four and two branes, respectively, with $M_1 = 6$, $M_2 = 4$ and $M_3 = 2$. The fluxes on the three stacks are given by

$$\begin{aligned}
(m_i^{(1)}, n_i^{(1)}) &= (1,1), (1,1), (-1,1)\ , \\
(m_i^{(2)}, n_i^{(2)}) &= (-1,1), (-2,2), (1,1)\ , \\
(m_i^{(3)}, n_i^{(3)}) &= (-1,1), (-1,1), (1,1)\ ,
\end{aligned} \tag{4.19}$$

which correspond to supersymmetric fluxes with similar solutions for the internal volumes as in the previous examples. The resulting gauge group is $[\mathbf{U(3)} \otimes \mathbf{U(2)} \otimes \mathbf{U(1)}]_1 \otimes [\mathbf{U(3)} \otimes \mathbf{U(2)} \otimes \mathbf{U(1)}]_2$ and the massless spectrum contains

Multiplicity	**Representation**
1	$(\mathbf{3},\mathbf{1},\mathbf{1};\bar{\mathbf{3}},\mathbf{1}\ ,\mathbf{1}) + (\bar{\mathbf{3}},\mathbf{1},\mathbf{1}\ ;\mathbf{3},\mathbf{1}\ ,\mathbf{1}) +$
	$2\times(\mathbf{1},\mathbf{2}\ ,\mathbf{1}\ ;\mathbf{1},\mathbf{2}\ ,\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{1},\mathbf{1})\ ,$
8	$(\mathbf{3},\mathbf{1}\ ,\mathbf{1}\ ;\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}\ ;\mathbf{3},\mathbf{1}\ ,\mathbf{1})\ ,$
4	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2}\ ,\mathbf{1};\bar{\mathbf{3}},\mathbf{1},\mathbf{1})\ ,$
12	$(\mathbf{1},\mathbf{2}\ ,\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\ \mathbf{1}\ ;\mathbf{1},\mathbf{2}\ ,\mathbf{1})\ ,$
2	$(\mathbf{1},\mathbf{3}\ ,\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1}\ ,\mathbf{1}\ ;\mathbf{1}\ ,\mathbf{3},\mathbf{1})\ ,$
36	$(\mathbf{1},\mathbf{1},\mathbf{1}\ ;\mathbf{1},\mathbf{1},\mathbf{1})\ ,$ (4.20)

where the 38 singlets have various $U(1)$ charges not explicitly displayed in (4.20). The model has therefore two copies of the Standard Model gauge group with four generations of quarks and leptons, eight Higgs multiplets plus two exotic states in the symmetric representations of $\mathbf{U(2)}_i$. The (non-chiral) states in the byfundamentals of $\mathbf{U(3)_1} \otimes \mathbf{U(3)}_2$, $\mathbf{U(2)_1} \otimes \mathbf{U(2)}_2$, $\mathbf{U(1)_1} \otimes \mathbf{U(1)}_2$ in the first two lines of (4.20), if given a vev, would break the gauge group to the diagonal Standard Model gauge group $\mathbf{U(3)} \otimes \mathbf{U(2)} \otimes \mathbf{U(1)}$. An alternative channel breaks the gauge symmetry down to $\mathbf{U(3)}_1 \otimes \mathbf{U(2)}_2 \otimes \mathbf{U(1)}_1$. The gauge symmetry breaking generates masses for various states. It is beyond the scope of

the present paper to study in detail the resulting models, but it is encouraging to find in a rather simple way the Standard Model gauge group and the representations which correspond to the quarks, leptons and Higgs multiplets.

A chiral model in the $Z_2 \times Z_2$ orientifold without discrete torsion

Let us consider again the four dimensional compactification on $(T^2)^3$ of type IIB theory orbifolded by the group $Z_2 \times Z_2$ and without discrete torsion ($\epsilon_i = 1,\ i = 1, 2, 3$). Let us add three stacks of magnetized D9 branes with $M_1 = 8,\ M_2 = 4,\ M_3 = 2$ and 32 $D5_1$ branes without magnetic fluxes. The supersymmetric magnetic fluxes are given by:

$$\begin{aligned}
(m_i^{(1)}, n_i^{(1)}) &= (-1,1),(1,1),(1,1)\ , \\
(m_i^{(2)}, n_i^{(2)}) &= (-1,1),(1,1),(1,1)\ , \\
(m_i^{(3)}, n_i^{(3)}) &= (-2,2),(1,1,),(1,1)\ .
\end{aligned} \tag{4.21}$$

The gauge group of this model is $[\mathbf{U(4)}]_{9_1} \times [\mathbf{U(2)}]_{9_2} \times [\mathbf{U(1)}]_{9_3} \times [\mathbf{USp(16)}]_5$. The chiral spectrum contains, in addition to three chirals in the adjoint of every gauge factor, Weyl fermions in the $D9_i\ -\ D9_j$ intersections in the following representations:

$$\begin{aligned}
&9_1 - 9_1 \ :\ 8 \times (\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})\ \ ,\quad 9_{2,3} - 9_{2,3}\ :\ 8 \times (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})\ , \\
&9_1 - 9_2 \ :\ 8 \times (\mathbf{4},\mathbf{2},\mathbf{1},\mathbf{1})\ \ ,\quad 9_1 - 9_3\ :\ 16 \times (\mathbf{4},\mathbf{1},\mathbf{1},\mathbf{1})\ , \\
&9_2 - 9_3 \ :\ 16 \times (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})
\end{aligned} \tag{4.22}$$

and fermions in the $D9_i\ -\ D5$ intersections:

$$\begin{aligned}
9_1 - 5 &:\ (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{16}) + (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},\bar{\mathbf{16}})\ , \\
9_2 - 5 &:\ (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{16}) + (\mathbf{1},\mathbf{2},\mathbf{1},\bar{\mathbf{16}})\ , \\
9_3 - 5 &:\ 2 \times [(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{16}) + (\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{16}})]\ .
\end{aligned} \tag{4.23}$$

The $9_1 - 9_3$ states, if given an appropriate vev, would break $\mathbf{U(4)} \to \mathbf{U(3)}$ leading to a standard model gauge group with eight generations. In order for the D5 gauge group to play the role of a hidden sector, the $D9_i - D5$ states have to be given a mass by adding, for example, appropriate Wilson lines.

5. A new orientifold of Scherk-Schwarz compactifications and non-BPS orientifold planes

The non-BPS D3 brane of Type I maps, after six T-dualities, into a D9 brane in the Type IIB orientifold defined by the orientifold projection $\Omega' = \Omega I_6 (-1)^{F_L}$, where I_6 denotes six parities in the internal space and $(-1)^{F_L}$ is the left spacetime fermion number. In this section we will construct a new orientifold model in nine dimensions in which such branes appear as D9 branes wrapped on the circle, with a Scherk-Schwarz type deformation of their spectrum. The model contains also a new type of non-BPS orientifold planes which have different couplings than the known non-BPS D branes in type II orientifolds. We believe that this model, which is the fourth type of nine-dimensional orientifold of Scherk-Schwarz type II strings, following the three previous ones [19], [20], is also the last possible construction in nine dimensions. The construction starts from the torus amplitude of Scherk-Schwarz compactifications [21, 22]

$$\mathcal{T} = \int \frac{d^2\tau}{\tau_2^6} \ \big[(|V_8|^2 + |S_8|^2) \Lambda_{m,2n} - (V_8 \bar{S}_8 + S_8 \bar{V}_8) \Lambda_{m+1/2,2n} + (|O_8|^2 + |C_8|^2) \Lambda_{m,2n+1} - (O_8 \bar{C}_8 + C_8 \bar{O}_8) \Lambda_{m+1/2,2n+1} \big] \ \frac{1}{|\eta|^{16}} \ , \tag{5.1}$$

where the notations are defined, for example, in the last two references in [15]. The first orientifold projection of Scherk-Schwarz type II strings [23, 19] is the standard left-right exchange $\Omega_1 = \Omega$, the second one [19] is based on $\Omega_2 = \Omega I_1$, where I_1 is the parity in the Scherk-Schwarz coordinate and generates a supersymmetry breaking perpendicular to the brane worldvolume, whereas the third one [20], which has the virtue of eliminating the closed string tachyon (and also the open string tachyon, in the sense of requiring a net number of 16 D8 branes by the RR tadpole conditions), is based on $\Omega_3 = \Omega I_1 (-1)^{f_L}$, where $(-1)^{f_L}$ is the left worldsheet fermion number.

The present construction starts from the orientifold projection $\Omega_4 = \Omega \ \delta \ (-1)^{F_L}$, where δ is the shift $X_9 \rightarrow X_9 + \pi R$. This is indeed a symmetry of the theory, provided that the shift and $(-1)^{F_L}$ are combined together. The Klein bottle amplitude

$$\mathcal{K} = \frac{1}{2} \int_0^\infty d\tau_2 \ (V_8 + S_8) \ (-1)^m P_m \ \frac{1}{\eta^8(2i\tau_2)} \tag{5.2}$$

has the peculiarity of symmetrizing the RR sector by projecting out the two form, whereas keeping the zero form and the four form. The BPS branes of this orientifold are therefore D3

and D7 branes, whereas all the other branes, including the top D9 branes we will describe in a moment, are non-BPS. The consistency of the construction can also be checked by performing the S-transformation into the tree-level closed channel

$$\tilde{\mathcal{K}} = \frac{2^5}{2} \int_0^\infty dl \ (O_8 - C_8) \ W_{2n+1} \ \frac{1}{\eta^8(il)} \ , \tag{5.3}$$

which reveal the presence of a new type of non-BPS O-planes, with no tension and RR charge, coupling only to the closed string tachyon and to a massive RR 10 form. The model does not demand by RR tadpole conditions the addition of D9 branes. However, they can be added consistently with the string constraints : particle interpretation and open-closed string duality. The open string amplitudes are

$$\mathcal{A} = \int_0^\infty \frac{dt}{t^{11/2}} \left[N\bar{N} \ (V_8 P_m - S_8 P_{m+1/2}) + \frac{N^2+\bar{N}^2}{2} \ (O_8 P_m - C_8 P_{m+1/2}) \right] \frac{1}{\eta^8(it/2)} \ ,$$

$$\mathcal{M} = \int_0^\infty \frac{dt}{t^{11/2}} (-1)^m \left[\pm \frac{(N+\bar{N})}{2} \ P_m \ \hat{O}_8 \pm \frac{(N-\bar{N})}{2} \ P_{m+1/2} \ \hat{C}_8 \right] \frac{1}{\hat{\eta}^8(it/2+1/2)} \tag{5.4}$$

and define consistent non-BPS D9 branes with gauge group $\mathbf{U(N)}$. The four different possible signs in the Mobius amplitude define the four different signs of the O9 couplings $(\pm,\pm)$ to the would-be closed tachyon and RR 10 form. Let us concentrate for simplicity on the $(-,+)$ sign. Then the spectrum of the model also contains open tachyons in the antisymmetric representations $\mathbf{N(N-1)/2 + \bar{N}(\bar{N}-1)/2}$ and (Majorana in nine dimensions) fermions in $\mathbf{N(N-1)/2 + \bar{N}(\bar{N}+1)/2}$. Since the fermions have KK masses shifted by 1/2, the spectrum can clearly be interpreted as a Scherk-Schwarz deformation of certain non-BPS branes with the same spectrum as the non-BPS D7 branes in the type I strings. The notable difference in the present case, however, is that the single brane case $N=1$ is stable, since the open string tachyons disappear. In the case of the single D7 brane in Type I string, the same phenomenon happens but the D7 brane is still unstable due to the tachyons coming from the D7-D9 interactions.

Similarly, the choice $(+,-)$ gives the Scherk-Schwarz deformed spectrum of the non-BPS D3 brane of Type I strings. In this case, the D9 brane is unstable due to the open string tachyon.

Since the RR sector contains a zero-form, a four-form and an eight-form, the model contains charged D3 and D7 branes. Let us, for concreteness, discuss the case of D7 branes. The D7-D7 and D7-O9 string amplitudes in the open channel are given by

$$\mathcal{A}_{77} = \frac{M^2}{2} \int_0^\infty \frac{dt}{t^{9/2}} \{ (V_6 O_2 + O_2 V_6) P_m - (S_6 S_2 + C_6 C_2) P_{m+1/2} \} \ \frac{1}{\eta^8} \ , \tag{5.5}$$

$$\mathcal{M}_{79} = -\frac{M}{2}\int_0^\infty \frac{dt}{t^{9/2}}\{(\hat{O}_6\hat{V}_2 - \hat{V}_6\hat{O}_2)(-1)^m P_m + (\hat{S}_6\hat{S}_2 - \hat{C}_6\hat{C}_2)(-1)^m P_{m+1/2}\}\ \frac{1}{\eta^8}\ .$$

The gauge group for M coincident D7 branes is therefore $\mathbf{USp(M)}$ and the open spectrum includes two massless scalars in the antisymmetric representation $\mathbf{M(M+1)/2}$, while the fermions are massive due to the Scherk-Schwarz deformation. The whole open string spectrum (5.5) is manifestly a soft Scherk-Schwarz deformation of a supersymmetric spectrum, whereas the closed string spectrum, due to the orientifold projection leading to the Klein bottle (5.2), is manifestly non-supersymmetric.

The amplitudes in the closed, tree-level channel

$$\tilde{\mathcal{A}}_{77} = \frac{M^2}{2^5}\int_0^\infty \frac{dl}{l}\{(V_6O_2 + O_2V_6 - S_6S_2 - C_6C_2)W_{2n} + (O_6O_2 + V_6V_2 - S_6C_2 - C_6S_2)W_{2n+1}\}\frac{1}{\eta^8}\ ,$$

$$\tilde{\mathcal{M}}_{79} = M\int_0^\infty \frac{dl}{l}\{(\hat{O}_6\hat{O}_2 + \hat{V}_6\hat{V}_2) - (-1)^n(\hat{S}_6\hat{C}_2 - \hat{C}_6\hat{S}_2)\}\ W_{2n+1}\frac{1}{\eta^8(il+1/2)}\ , \qquad (5.6)$$

reveal couplings of the charged D7 branes with the dilaton and the RR eight-form and also couplings to the would-be tachyon and to the massive RR eight-form coming from the "twisted" sector.

A similar analysis for D coincident charged D3 branes gives a gauge group $\mathbf{SO(D)}$.

The peculiarity of the present model is the nature of supersymmetry breaking in the closed and in the open sector, in the case with O9 and charged D7 branes. Whereas the D7-D7 brane interactions are clearly a soft deformation of supersymmetric ones, for the closed spectrum and the D7-O9 interactions, due to the orientifold projection Ω_4, reflected in the Klein bottle amplitude, the softness manifests itself differently. Level by level supersymmetry seems to be badly broken, but the breaking becomes soft when including the whole spectrum. Indeed, the Klein and Mobius amplitudes actually vanish in the $R \to \infty$ limit, as appropriate for a soft supersymmetry breaking spectrum, displaying a kind of zig-zag supersymmetry providing cancellations between different mass levels.

6. Conclusions

We showed the complete equivalence between BPS and non-BPS D-branes with appropriately mapped internal magnetic fluxes in type I orientifold models. We discussed some necessary conditions obtained by requiring that one supersymmetry be preserved and

the tadpole conditions to be fulfilled, excluding in particular toroidal and some orbifold constructions of this type.

Our main goal in this paper was to reconsider some class of models featuring the phenomenon of "brane supersymmetry breaking " [9], in which exotic O-planes of positive tension and charge did force the introduction of antibranes, which did break supersymmetry. As anticipated in [11] and explicitly showed in Section 4, this conclusion can be avoided for particular models with magnetized D9 branes and negatively induced tension in one internal torus. Whereas the non-chiral constructions in [11] used bulk branes, we did construct explicit chiral supersymmetric examples with branes at orbifold fixed points (fractional branes). A novelty here is the appearance of new constraints from twisted RR tadpole conditions due to internal magnetic fields which generate physical four-dimensional couplings of D-branes to twisted RR fields. If our explicit examples are not fully realistic, one could reasonably expect that phenomenologically quasi-realistic models along these lines can be constructed, eventually combining the present constructions with recent approaches to moduli stabilization [24]. Since in most of our constructions all tadpole conditions were satisfied with only magnetized D9 branes, it is also plausible that some of these constructions have $Z_2 \times Z_2$ heterotic duals [25].

Finally, we did present a new Scherk-Schwarz orientifold of type II strings (in addition to the three already existing ones in the literature) which involves a new type of non-BPS O9 orientifold planes, coupling to the closed string tachyon. One of the possible future lines of investigation in such models is related to the role of the couplings to the O-planes of the would-be closed tachyon for the tachyon dynamics. Indeed, in the regime where the corresponding closed string scalar becomes more and more tachyonic, its couplings to the O-planes become more and more important and cannot be neglected. The soft nature of supersymmetry breaking in this new orientifold construction is realized in an interesting way, with the O9 plane disappearing in the limit of supersymmetry restoration.

7. Appendix

A useful Riemann identity for Jacobi functions is

$$\sum_{\alpha,\beta=0,1/2} \eta_{\alpha,\beta}\ \vartheta[{\alpha \atop \beta}](z)\ \prod_{i=1}^{3}\ \vartheta[{\alpha \atop \beta+v_i}] =$$

$$-2\ \vartheta_1(-\frac{z}{2})\ \vartheta_1(\frac{z-v_1+v_2+v_3}{2})\ \vartheta_1(\frac{z+v_1-v_2+v_3}{2})\ \vartheta_1(\frac{z+v_1+v_2-v_3}{2})\ , \quad (7.1)$$

valid for $v_1+v_2+v_3=0$.

A notation used very often in the text is

$$(V_8-S_8)(z_i)\ \equiv\ \sum_{\alpha\beta}\eta_{\alpha\beta}\ \vartheta[{\alpha \atop \beta}](0)\ \prod_{i=1}^{3}\ \vartheta[{\alpha \atop \beta-z_i}]\ , \quad (7.2)$$

with similar notations for other parts of the amplitudes with open string propagations.

In all string amplitudes written in the text, we did not explicitly write the contributions, proportional to $1/(4\pi^2\alpha')^{d/2}$ coming from the integral over the non-compact momenta, where d is the number of non-compact dimensions.

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